Read Mapping (1)

> Peter N. Robinson

Basics

Review

# Read Mapping de Novo Assembly

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Genomics: Lecture #2 WS 2014/2015

# Today

#### Read Mapping (1)

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#### Basics

Review

- Genome assembly: the basics
- Hamiltonian and Eulerian Graphs: Review
- De Bruijn graphs: Basics
- De Bruijn graphs for genome assembly: Simplified
- De Briun graphs for genome assembly: Simple pair end

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• De Bruijn graphs: real life



### Genome assembly: the basics

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Basics

Review

The process of puzzling together a complete genome sequence of an organism for which shotgun sequencing has been performed is referred to as **genome assembly**.

- As the costs for sequencing have declined, the major challenge becomes computational
- Can we sequence and de novo assemble a large (> 100 Mb) genome with the short (50-250bp) reads typical of current NGS protocols?

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### Genome assembly: the basics



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Basics

Review

- There are two major classes of assembly algorithms
  - Overlap-layout consensus (OLC)
     De bruijn graph (DBG)
- OLC was widely used back in the day when sequencing was performed by the low-throughput, longer-read Sanger method.
- DBG based methods have dominated the scene since the introduction of NGS

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Basics

Review

To get intuition about genome assembly, let us consider a **idealized genome** that represents a long random sequence of four bases and that does not contain repeats or other complex structures.

- Consider the simplest sequencing strategy: single-end, whole-genome shotgun (WGS).
- That is, we sample equal-length fragments with starting points randomly distributed across the genome

• For now, ignore sequencing errors and biases

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Basics

Review

- Thus, our shotgun sequencing can be likened to a process that samples bases from all genome positions at random
- The chance that any particular base is sampled is very low in a single sampling process

• However, we perform the sampling process a very large number of times

Any suggestions as to how we might model this?

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Basics

Review

The Poisson distribution expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events are iid.

$$f(k;\lambda) = P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$
(1)

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- k refers to number of reads that overlap a certain genomic position ("coverage")
- $\lambda$  mean sequencing depth

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Basics

Review

Let's look at the model more closely

- G: genome size (e.g.,  $3.2 \times 10^9$  nucleotides for humans)
- L: read length (e.g., 100 nucleotides for a typical Illumina run)
- N: read number
- n<sub>b</sub>: total number of sequenced bases

It is now easy to calculate that

$$n_b = N \times L \tag{2}$$

• Similarly, the average coverage depth per base is  $d_b = \frac{n_b}{c}$ 

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Basics

Review

**k-mers**: subsequences with k nucleotides (here: k=3)

• Consider the following k-mers in a small genome of 17 nucleotides

Composition, (TCATTCTTCAGGTCAAA) TCA CAT АТТ TTC TCT СТТ TTC TCA CAG AGG GGT GTC TCA CAA ΔΔΔ

● How many 3-mers are there in this genome?

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Basics

Review

- In general there are L − k + 1 k-mer subsequences in a sequence of length L with k ≤ L.
- Let's say we want to know the total number of k-mers in our WGS data. Since we have N reads, each of which has L k + 1 k-mer subsequences, the total number of k-mers (nk) is

$$n_k = N \times (L - k + 1) \tag{3}$$

- The coverage depth for k-mers is then  $d_k = \frac{n_k}{C}$ .
- The ratio between the coverage depth for bases and that for k-mers is then

$$\frac{d_b}{d_k} = \frac{n_b/G}{n_k/G} = \frac{L}{L-k+1}$$
(4)

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Basics

Review

Say we are performing de novo sequencing for an organism that has not been sequenced before. How can we estimate its overall genome size?

- The number of k-mers in the WGS reads (*n<sub>k</sub>*) can be directly counted
- The mean coverage depth of k-mers can be estimated from the peak value of the empirical k-mer coverage depth distribution curve



Graphic: Zhenyu Li et al., Briefings in functional genomics (2012) 11 (1): 25-37.

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Basics

Review

With this data in hand, we can now estimate the genome size as

$$G \approx \frac{n_k}{d_k}$$
 (5)

and we can estimate the actual base coverage by

$$d_b \approx \frac{L}{L-k+1} \times d_k \tag{6}$$

- Here, we would use a value of k such that we do not expect to see a given k-mer more than once in a random genome
- In practice, these estimates are not exact even in a random genome because of sequencing errors (why?).

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Basics

Review

Getting back to our initial question, we can now use the mean base coverage estimate  $\lambda = d_b$  to estimate the probability that a given base will not be covered

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$
(7)

Therefore, the probability of seeing at least one read at a given position is

$$P(X > 0) = 1 - e^{-\lambda} \tag{8}$$

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Basics

Review

• So if we want to estimate the mean read depth required such that at least 99% of the genome is covered once (and thus the probability of any base is at least 99%)<sup>1</sup>, we have

$$egin{array}{rcl} P(X>0) = 0.99 &=& 1-e^{-\lambda} \ e^{-\lambda} &=& 0.01 \ -\lambda &=& -4.605 \end{array}$$

- Thus we need to sequence to an average depth of at least 4.6 to get at least 99% of the genome covered at least once.
- This roughly explains the goal of 6x coverage in initial Sanger sequencing projects of the human genome

<sup>&</sup>lt;sup>1</sup>linearity of expectation

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Basics

Review

Now let us consider **contigs**: combinations of overlapping reads that represent contiguous sequence



- Collection of N = 21 reads assembled into 6 contigs
- The contigs are assumed to be the best possible representation of the original DNA sequence
- Note that the actual locations of the contigs and their orientation to one another are unknown to us.

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Basics

Review

The initial steps of genome assembly are basically an attempt to find contigs



Genome: 3.2 Gb Many copies of genome



Reads: 500bp Only one end sequenced Not all fragments sequenced

Find overlapping reads

tgctgctcctacaacatcggccgtgcctg atcggccgtgcctggaataagccct

...tgctgctcctacaacatcggccgtgcctggaataagccct...

Merge overlapping reads into contigs



Result of assembly is set of contigs with gaps

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Basics

Review



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 Competition to assemble the human genome: whole-genome shotgun vs. BAC by BAC

# The human genome project(s)

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Basics

Review

Public effort: Lander et al., Nature, Feb. 15, 2001.

- US, UK, France, Germany, Japan, China
- government labs and universities.
- BAC-by-BAC sequencing.

Commercial: Venter et al., Science, Feb. 16, 2001.

• whole genome random shotgun sequencing.

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• Celera (www.celera.com)

# BAC by BAC

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Basics

Review

- BAC: bacterial artificial chromosome
- BACs have inserts of 100,000–300,000 nucleotides
- Do shotgun sequencing on each separate BAC;
- BACs are much smaller than the human genome and correspondingly easier to assemble.
- First assemble individual BACs, then fit overlapping BACs together



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Advantage: highly accurate. Disadvantage: Slow

# Whole genome shotgun



Myers EW (2000) A Whole-Genome Assembly of Drosophila Science 287:2196-2204

- All against all pairwise alignment
- Merge to contigs if overlap big enough
- Nicknames for contigs: small = rock, smaller = stone, smaller = pebble.

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# Whole genome shotgun



• Additional processing to piece together contigs

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# Whole genome shotgun: Overlaps

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Basics

Review

- We will be covering mainly algorithms for creating contigs from reads in these lectures
- Let us begin with another topic to build intuition: How much **overlap** do we need?
- Key questions: How many contigs are there? How big are the gaps? How long are the contigs?



Overlap between communism and capitalism

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# Whole genome shotgun: Contigs and Overlaps

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Basics

Review

Recall our definitions

- G: genome size. Assume  $3 \times 10^9$  nucleotides
- L: read length. Assume 500 nucleotides
- N: read number
- $n_b = N \cdot L$ : total number of sequenced bases
- $\lambda = NL/G$  is the coverage

For instance, 10x coverage of the human genomes requires

$$N = \lambda G/L = 10 \cdot 3 \times 10^9/500 = 60$$
 million reads

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### Reads: Probability to start at a given base

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Basics

Review

- In a genome of length *G*, a read of length *L* can start anywhere except at the very ends of the chromosomes
- In humans there are c = 23 chromosomes, so  $c \times (L-1)$  positions cannot represent start positions.
- For L = 500, we have  $23 \times 499 = 11477$  such positions, but these can be ignored in a genome of  $3 \times 10^9$  nucleotides
- Thus, the probability that a read starts at base *i* is well approximated by  $P(\text{read starts at } i) \approx N/G$  if there are a total of N reads

### Reads: Probability to start in an interval

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Basics

Review

- Consider an interval *I* that is as long as a read (*L* nucleotides).
- The expected number of reads that start in I is then  $\lambda = L \times N/G$ .
- Assuming a Poisson distribution, the probability that no read starts in *I* is then

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$
(9)

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• The probability that at least one read starts in I is then

$$P(X > 0) = 1 - P(X = 0) = 1 - e^{-\lambda}$$
 (10)

# Reads: Probability to start in an interval

Read Mapping (1)

> Peter N. Robinson

Basics

Review

- Consider a nucleotide at position *i*
- This nucleotide is in a gap between contigs if no read starts in the interval

$$[i - L + 1, i]$$

- This interval has length L, and thus, the probability that no read starts in it is  $e^{-\lambda}$
- By linearity of expectation, we can estimate the number of nucleotides in gaps across the entire assembly as

$$G \cdot e^{-\lambda}$$

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• Correspondingly, the number of nucleotides included in contigs is  $G\cdot \left(1-e^{-\lambda}
ight)$ 

# Contigs: How many are there?



- Each contig has a unique rightmost read ("R")
- The probability that a given read is the rightmost read is the same as the probability that no other read starts within the read
- If the read starts at position *i*, this is the probability that no read starts within the interval [i - L + 1, i], which we have already calculated as  $e^{-\lambda}$

# Contigs: How many are there?

#### Read Mapping (1)

#### Peter N. Robinson

#### Basics

Review

- The number of contigs must be equal to the number of rightmost reads
- There are a total of N reads, each of which has a probability of  $e^{-\lambda}$  of being an R reads. Thus, the expected number of contigs is

 $Ne^{-\lambda}$ 

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• The expected number of reads per contig is then  $1/e^{-\lambda}$ 

# Contigs: How big are they?

Read Mapping (1)

> Peter N. Robinson

Basics

Review

- We have seen that the expected size of the sequenced region of the genome is  $(1 e^{-\lambda}) \cdot G$
- The expected number of contigs is  $Ne^{-\lambda}$  Therefore, the expected size of a contig is simply

$$\frac{(1-e^{-\lambda})\cdot \textit{G}}{\textit{N}e^{-\lambda}}$$

- Thus if we go for a coverage of  $\lambda = 6$  of the human genome with 500 nt reads, we would expect roughly
  - $N = \lambda G/L = 36$  million reads
  - 2  $100\% \times (1 e^{-\lambda}) = 99.8\%$  of the genome being sequenced

**③** A total of  $Ne^{-\lambda}$ =89,235 contigs

• An average contig length of  $\frac{(1 - e^{-\lambda}) \cdot G}{Ne^{-\lambda}} = 33,536$  nucleotides

# **Contigs and overlaps**

Read Mapping (1)

> Peter N. Robinson

Basics

Review

But we have completely neglected the topic of **how much of an overlap** is required to connect two reads?

- Let say we require an overlap of one nucleotide only
- Then any two random reads will overlap with a probability of 1/4 – not exactly what we want...
- Let  $\theta$  refer to the proportion of L required to detect an overlap



• We will now combine a group of reads to a contig if they are connected by overlaps of length  $\geq \theta L$ 

## Expected number of contigs

Read Mapping (1)

> Peter N. Robinson

Basics

Review

Let us now calculate the expected number of contigs, given that we demand an overlap of at least  $\theta L$  between combined reads

- , As before the probability that a read starts at a given position is  ${\cal N}/{\cal G}$
- The probability that k reads start in an interval that is L long is again approximated by the Poisson
- The calculation that a given read at posistion *i* is the rightmost read now requires not that there is no read in the interval [i L + 1, i], but instead that there is no read in the leftmost  $(1 \theta)$  proportion of this interval



- We need to calculate the probability that zero reads start in  $(1 \theta)L$ .
- Above, the expected number of reads that start in *I* of length *L* is then λ = L × N/G.
- Here, we adjust this to reflect the expected number of reads that start in  $(1 \theta)L$  to be  $(1 \theta)\lambda$ .

# Expected number of contigs

Read Mapping (1)

> Peter N. Robinson

Basics

Review



• The expected number of contigs is then N (number of reads) time the probability that a read is the rightmost read of an island, which is equivalent to their being no reads starting in  $(1 - \theta)L$ 

$$\mathbb{E}[\#\text{contigs}] = N \times P(\text{no read starts in } (1-\theta)L)$$
  
=  $Ne^{-(1-\theta)\lambda}$   
=  $Ne^{-(1-\theta)LN/G}$  by definition of  $\lambda$ 

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### Expected number of contigs



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	Outline
Read Mapping (1)	
Peter N. Robinson	
Basics	
Review	<b>1</b> Genome assembly: the basics

**2** Hamiltonian and Eulerian Graphs: Review
### Königsberg

Read Mapping (1)

> Peter N. Robinson

Basics

Review

The "Hello World" of Eulerian graphs is of course Königsberg with its seven bridges. Königsberg is located on both sides of the Pregel River, and comprises two large islands which were connected to each other and the mainland by seven bridges.



### Königsberg

#### Read Mapping (1)

Peter N. Robinson

Basics

Review

The problem was to find a walk through the city that would cross each bridge once and only once.

• Euler formulated the problem as a graph problem





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### **Genome Sequencing And Graphs**

#### Read Mapping (1)

Peter N. Robinson

Basics

Review

Our goal today is to find an algorithm that will allow us to take a collection of **short NGS sequence reads** – say, strings of 100–250 nucleotides in length with the lettersd ACGT – and to output a longer string representing the **Genome** that was sequenced.

• We will present several simplified scenarios with the goal of motivating and explaining the de Bruijn graph and its use in genome assembly algorithms.

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Read Mapping (1)

> Peter N. Robinson

Basics

Review

We will begin by discussing a ridiculously naive string reconstruction problem. Here, and in the following, the "string" will represent a genome that we have sequenced, and the k-mer subsequences (with k=3) will represent our short reads.

• We will begin by examining a small genome of 17 nucleotides

# **TCATTCTTCAGGTCAAA**

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Read Mapping (1)

> Peter N. Robinson

Basics

Review

Imagine we have a function called  $composition_k$  that takes a DNA sequence and returns a set of all k-mers contained in it

• In the following examples we will choose k = 3

Composition<sub>k</sub>(TCATTCTTCAGGTCAAA) TCA

CAT ATT

TTC TCT

> CTT TTC

TCA

CAG

AGG

GGT GTC

TCA

CAA

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Read Mapping (1)

> Peter N. Robinson

Basics

Review

Although we can display the k-mers in "genome order", which would make it ridiculously easy to reconstruct the original genome, in actuality we do not know the original order of the kmers. Therefore, we might as well show them lexicographically.

```
Composition<sub>k</sub>(TCATTCTTCAGGTCAAA)

"Genome order":

TCA CAT ATT TTC TCT CTT TTC TCA CAG AGG GGT GTC TCA CAA AAA

"Lexicographic order"

AAA AGG ATT CAA CAG CAT CTT GGT GTC TCA TCA TCA TCT TTC TTC
```

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### Naive Genome Assembly Read Mapping (1) Peter N. Robinson Let us now put each of the k-mers into the node of a graph and connect the graph by edges Review Composition, (TCATTCTTCAGGTCAAA) "Genome order": TCA CAT ATT TTC TCT CTT TTC TCA CAG AGG GGT GTC TCA CAA AAA Put k-mers into nodes TCA Connect nodes with edges

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Read Mapping (1)

> Peter N. Robinson

Basics

Review

But what if the nodes are not connected? Can we order them and put them back together again?

TCATTCTTCAGGTCAAA



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Read Mapping (1)

> Peter N. Robinson

Basics

Review

The question is whether we can reconstruct the original string if we only have the nodes and do not know what order they are on?

• Challenge: Find the following sequence based only on a collection of 3-mer subsequences:

# TCATTCTTCAGGTCAAA

- The basic strategy to do this involves searching for overlaps between k-mers.
- E.g., connect k-mer<sub>i</sub> with k-mer<sub>j</sub> if

$$suffix(k-mer_i) = prefix(k-mer_j)$$





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#### Read Mapping (1)

Peter N. Robinson

Basics

Review

However, just to demonstrate how to generate a path that represents the sequence, let us pretend we are omnisicent start with the k-mer TCA and connect it to CAT and ATT.

TCATTCTTCAGGTCAAA



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TCATT

	Naive Genome Assembly
Read Mapping (1) Peter N. Robinson	
Basics Review	Conitnuing in this way
	AAA AGG ATT CAA CAG CAT CTT GGT GTC TCA TCA TCA TTC TTC TTC TCATTCT

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#### **Naive Genome Assembly** Read Mapping (1) Peter N. Robinson Further ... Review TCATTCTTCAGGTCAAA (CAT) CAG TCA CAA (CTT GTC TCA TCA TCT TTC TTC AAA AGG TCATTCTTCAG

	Naive Genome Assembly
Read Mapping (1) Peter N.	
Robinson Basics Review	And finally
	AAA AGG ATT CAA CAG CAT CTT GGT GTC TCA TCA TCA TTT TTC TTC TCATTCTTCAGGTCAAA

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### **Naive Genome Assembly** Read Mapping (1) Peter N. Robinson Notice that the solution to our problem was a path that visited every node exactly once. Review TCATTCTTCAG GTCAAA CAT TCA TCA TCA тст

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TCATTCTTCAGGTCAAA

# Hamiltonian path

Read Mapping (1)

> Peter N. Robinson

Basics

Review

- A **Hamiltonian path** is a path in an undirected or directed graph that visits each vertex exactly once. A Hamiltonian cycle is a Hamiltonian path that is a cycle.
  - Determining whether Hamiltonian paths and cycles exist in graphs is the Hamiltonian path problem, which is NP-complete.



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### Another approach

Read Mapping (1)

> Peter N. Robinson

Basics

Review

Instead of labeling the nodes with the k-mer subsequences, let us label the edges with these k-mers

TCATTCTTCAGGTCAAA

 TCA
 CAT
 ATT
 TTC
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### Another approach

#### Read Mapping (1)

Peter N. Robinson

#### Basics

Review

We will then label the nodes with the (k-1)-mer, i.e., 2-mer suffixes and prefixes



#### TCATTCTTCAGGTCAAA



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### Constructing a de Bruijn graph

Read Mapping (1)

> Peter N. Robinson

Basics

Review

Let us now merge identically labels nodes in this graph. We will show the steps along the way for our example graph. A key idea is that we will **merge identical nodes whilst retaining the edges**.

TCATTCTTCAGGTCAAA



### Constructing a de Bruijn graph Read Mapping (1) Peter N. Robinson Merge two CA nodes whilst retaining their edges Review **TCATTCTTCAGGTCAAA** GGT AGG AG

CAT

---- AT

TCA

TC

ATT

TTC

TC

TCT

CTT

TT

GTC

TTC 🔔 TCA

TC

CAG

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AA



# Constructing a de Bruijn graph



#### Constructing a de Bruijn graph Read Mapping (1) Peter N. Merge two TT nodes whilst retaining their edges. Continuing... Robinson **TCATTCTTCAGGTCAAA** Review GGT AGG AG GTC CAG AAA TCA \_ TCT СТ

CT

TCA

CAT

TC

TTC

CAA

### Constructing a de Bruijn graph



## Constructing a de Bruijn graph



#### Constructing a de Bruijn graph Read Mapping (1) Continuing... Peter N. Robinson **ΤCATTCTTCATCTTCTTCAGTCAAA** Review GGT AGG GT AG GTC CAG AAA TCT СТ TCA) TCA TC AA AA CA ----TCA CAA CTT TTC CAT TT( $\mathbf{TT}$ ATT AT

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# Constructing a de Bruijn graph



#### Traversing a de Bruijn graph Read Let us now examine how we might reconstruct the original Mapping (1) sequence from this de Bruijn graph Peter N. Robinson **TCATTCTTCAGGTCAAA** Review GGT AGG GT AG GTC CAG AA TCT CT TC AA CA CAA CT ТСАТТ...





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### Hamilton and Euler

#### Read Mapping (1)

Peter N. Robinson

Basics

Review

So we have seen two potential methodologies for traversing a graph to reconstruct a sequence based on the Hamiltonian and the Eulerian path problem.

TCATTCTTCAGGTCAAA

TCATTCTTCAGGTCAAA





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TCATT...

• Which do we take?

### Hamilton and Euler

#### Read Mapping (1)

Peter N. Robinson

Basics

Review

The **Eulerian path problem** (Is there a path that visits every edge exactly once), and the **Hamiltonian path problem** (is there a path that visits every edge exactly once?) are superficially similar.

- It turns out that<sup>2</sup>
  - the Eulerian path problem has efficient algorithms to solve it
  - the Hamiltonian path problem is NP complete

<sup>2</sup>We will not review proofs here, they are standard  $\rightarrow$   $\langle \Xi \rangle$   $\langle \Xi \rangle$   $\equiv$   $\neg \land \land$ 

## de Bruijn Graph of k-mers



#### Peter N. Robinson

Basics

Review

The de Bruijn graph of a collection of k-mers is

- A representation of every k-mer as an edge between its prefix and its suffix
- The nodes of the graph are thus the (k-1)-mer suffices and prefices
- All nodes with identical labels are merged, preserving edges

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### de Bruijn Graph of k-mers Read Mapping (1) Peter N. Robinson **Algorithm 1** Create annotation Review 1: Form a unique node for each k-mer in kmers 2: for each k-mer $\in$ kmers do 3: Connect prefix node and suffix node with edge 4: end for

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### **Eulerian Graphs**

Read Mapping (1)

> Peter N. Robinson

Basics

Review

A **Eulerian cycle** is a path that traverses every edge exactly once and returns at the end of the traversal to the start node.



• Does this graph contain a Eulerian cycle?
# **Eulerian Graphs**

Read Mapping (1)

> Peter N. Robinson

Basics

Review

A **Eulerian cycle** is a path that traverses every edge exactly once and returns at the end of the traversal to the start node.



• Does this graph contain a Eulerian cycle?

# **Eulerian Graphs**

Read Mapping (1)

> Peter N. Robinson

Basics

Review

A **Eulerian cycle** is a path that traverses every edge exactly once and returns at the end of the traversal to the start node.



• Does this graph contain a Eulerian cycle?

### N50



- Arrange the contigs from largest to smallest
- Find the position where the contigs cover 50% of the total genome size
- The length of the contig at this position is defined as the N50
- The longer the N50 is, the better the assembly

# Finally

Read Mapping (1)

> Peter N. Robinson

Basics

Review

- Email: peter.robinson@charite.de
- Office hours by appointment

#### Further reading

- Miller JR (2010) Assembly algorithms for next-generation sequencing data. Genomics 95:315–327
- Flicek P, Birney E (2009) Sense from sequence reads: methods for alignment and assembly. *Nature Methods* 6:S7–S11
- Li Z (2011) Comparison of the two major classes of assembly algorithms: overlap-layout-consensus and de-bruijn-graph.

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